5.2 – Properties of Exponents and Power Functions

Daily Objectives:

1. Introduce algebraic proof
2. Review the properties of exponents
3. Introduce the parent power function and distinguish it from the exponential function
4. Find solutions to power equations using properties of exponents
5. Introduce rational exponents as a means of solving equations

Investigation: Properties of Exponents

Part 1: Write each product in expanded form, and then rewrite in exponential form.

a. $2^3 \cdot 2^4$

$(2\cdot 2\cdot 2)\cdot(2\cdot 2\cdot 2\cdot 2)$

2^7

b. $x^2 \cdot x^5$

$x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x$

x^7

Product Property of Exponents: $a^m \cdot a^n =$ a^{m+n}

Part 2: Write the numerator and denominator of each quotient in expanded form. Reduce by eliminating common factors. Then rewrite the factors that remain in exponential form.

a. $\frac{4^5}{4^2} = \frac{\cancel{4}\cdot\cancel{4}\cdot 4\cdot 4\cdot 4}{\cancel{4}\cdot\cancel{4}} = 4^3$

b. $\frac{x^8}{x^6} = \frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}} = x^2$

Quotient Property of Exponents: $\frac{a^m}{a^n} =$ a^{m-n}

Part 3: Write each quotient in expanded form, reduce and rewrite in exponential form:

a. $\frac{2^2}{2^3} = \frac{\cancel{2}\cdot\cancel{2}}{\cancel{2}\cdot\cancel{2}\cdot 2} = \frac{1}{2} = 2^{-1}$

b. $\frac{x^3}{x^6} = \frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x\cdot x} = \frac{1}{x^3} = x^{-3}$

Rewrite each quotient using the property from Part 2.

a. 2^{-1}

b. x^{-3}

Definition of Negative Exponents: $\frac{1}{a^n} = \underline{a^{-n}}$

Step 4: Write the following expressions in expanded form. Then rewrite in exponential form:

a. $(3^2)^4$

$(3\cdot3)(3\cdot3)(3\cdot3)(3\cdot3)$

3^8

b. $(x^9)^3$

$(x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x)\cdot(x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x)(x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x)$

x^{27}

Power to **a** Power **Property:** $(a^n)^m = \underline{a^{mn}}$

Part 5: Write the following expressions in expanded form. Then rewrite in exponential form:

a. $(3x)^4$

$3x\cdot 3x\cdot 3x\cdot 3x$

$3\cdot3\cdot3\cdot3\cdot x\cdot x\cdot x\cdot x$

3^4x^4

b. $\left(\frac{x}{y}\right)^2$ $\frac{x}{y}\cdot\frac{x}{y} = \frac{x^2}{y^2}$

Power of a Product Property: $(ab)^m = \underline{a^m b^m}$

Power of a Quotient Property: $\left(\frac{a}{b}\right)^n = \underline{\frac{a^n}{b^n}}$

Step 6: (1) Write each quotient in expanded form. Then simplify. (2) Use the quotient property to rewrite each quotient in exponential form.

a. $\frac{4^3}{4^3}$ $\frac{\cancel{4}\cdot\cancel{4}\cdot\cancel{4}}{\cancel{4}\cdot\cancel{4}\cdot\cancel{4}} = 1$

$4^0 = 1$

b. $\frac{x^5}{x^5} = \frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}} = 1$

$x^0 = 1$

Zero Exponents: $a^0 =$ ___1___

Extension of Product of a Quotient Property: $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$

Power Property of Equality: If $a = b$, then $a^n = b^n$

Common Base Property of Equality: If $a^n = a^m$, and $a \neq 1$, then $n = m$

Example 1: Simplify the following expressions by using the properties of exponents.

a. $m^4 \cdot m^{10}$

m^{14}

b. $x^{-3} \cdot x^5$

x^2

c. x^{-8}

$\frac{1}{x^8}$

d. $\frac{x^9}{x^7}$ x^2

e. $(2a^4)^2$

2^2a^8

$4a^8$

f. $\left(\frac{x^4y^2}{xy^7}\right)$ $x^3y^{-5} = \frac{x^3}{y^5}$

g. $\left(\frac{y^3}{z^2}\right)^{-5}$

$\frac{y^{-15}}{z^{-10}} = \frac{z^{10}}{y^{15}}$

h. $\left(\frac{15a^0bc^{13}}{3b^4c^2}\right)^{-2}$

$\frac{15^{-2}a^0b^{-2}c^{-26}}{3^{-2}b^{-8}c^{-4}} = \frac{15^{-2}b^6c^{-22}}{3^{-2}}$

$\frac{3^2b^8c^4}{15^2b^2c^{26}}$ $\frac{3^2b^6}{15^2c^{22}}$

$\frac{9b^6c^{-22}}{225}$ $\frac{9b^6}{225c^{22}}$

$\frac{9b^6}{225c^{22}}$

Exponential Function

The general form of an exponential function is

$$y = ab^x$$

where a and b are constants and $b > 0$.

Power Function

The general form of a power function is

$$y = ax^n$$

where a and n are constants.

What is the difference between an exponential function and a power function?

In an exponential function, the variable is the x-value. In a power function the variable is the base of the power.

Example 2: Solve for positive values of x:

a. $x^4 = 3000$

$\sqrt[4]{x^4} = \sqrt[4]{3000}$

$x \approx 7.4008$

b. $6x^{2.5} = 90$

$\frac{6x^{2.5}}{6} = \frac{90}{6}$

$x^{2.5} = 15$

$\sqrt[2.5]{x^{2.5}} = \sqrt[2.5]{15}$

$x \approx 2.95418$

c. $x^2 + 12 = 48$

$-12 \quad -12$

$\sqrt{x^2} = \sqrt{36}$

$x = 6$

d. $2x^3 + 10x^3 = 972$

$\frac{12x^3}{12} = \frac{972}{12}$

$\sqrt[3]{x^3} = \sqrt[3]{81}$

$x \approx 4.32675$

Formative Assessment

a. $x^6 = 4{,}826{,}809$

$\sqrt[6]{x^6} = \sqrt[6]{4{,}826{,}809}$

$x = 13$

b. $5x^5 = 1{,}215$

$\frac{5x^5}{5} = \frac{1{,}215}{5}$

$\sqrt[5]{x^5} = \sqrt[5]{243}$

$x = 3$

c. $7x^2 + 22 = 1394$

$-22 \quad -22$

$\frac{7x^2}{7} = \frac{1372}{7}$

$\sqrt{x^2} = \sqrt{196}$

$x = 14$